Error propagation: *f = f(x)*

$$\overline{f(x)} \approx f(\bar{x})$$

$$\overline{f(x)} = \left\langle f(\bar{x}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_x + \ldots \right\rangle = f(\bar{x}) + \left\langle \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_x \right\rangle$$

$$\sigma_f = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \sigma_x$$

$$\sigma_f^2 = \left\langle \left(f(x) - \overline{f(x)}\right)^2 \right\rangle = \left\langle \left(f(\bar{x}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_x - \overline{f(x)} \right)^2 \right\rangle$$

$$= \left\langle \left(\left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_x \right)^2 \right\rangle = \left.\frac{\partial f}{\partial x}\right|_{\bar{x}}^2 \sigma_x^2$$

Example

$$f = x^n \sin(bx)$$

$$\sigma_f = \left|\frac{\partial f}{\partial x}\right| \sigma_x = \left| n x^{n-1} \sin(bx) + x^{n-1} b \cos(bx) \right|_{\bar{x}} \sigma_x$$

Relative error

$$f = ax^n$$

$$\frac{\sigma_f}{f} = \frac{n a x^{n-1} \sigma_x}{a x^n} = n \frac{\sigma_x}{x}$$

Error propagation: *f = f(x,y,z,…)*

$$\sigma_f^2 = \left|\frac{\partial f}{\partial x}\right|^2 \sigma_x^2 + \left|\frac{\partial f}{\partial y}\right|^2 \sigma_y^2 + \left|\frac{\partial f}{\partial z}\right|^2 \sigma_z^2 + \ldots$$

use $f(x,y) = f(\bar{x},\bar{y}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_x + \left.\frac{\partial f}{\partial y}\right|_{\bar{y}} \delta_y + O(\delta_x^2, \delta_y^2)$

and uncorrelated variables $\langle \delta_x \delta_y \rangle = 0$

Example $f = x + y$ $\qquad \sigma_f^2 = \sigma_x^2 + \sigma_y^2$

Example $g = \frac{4\pi l}{T^2}$

absolute

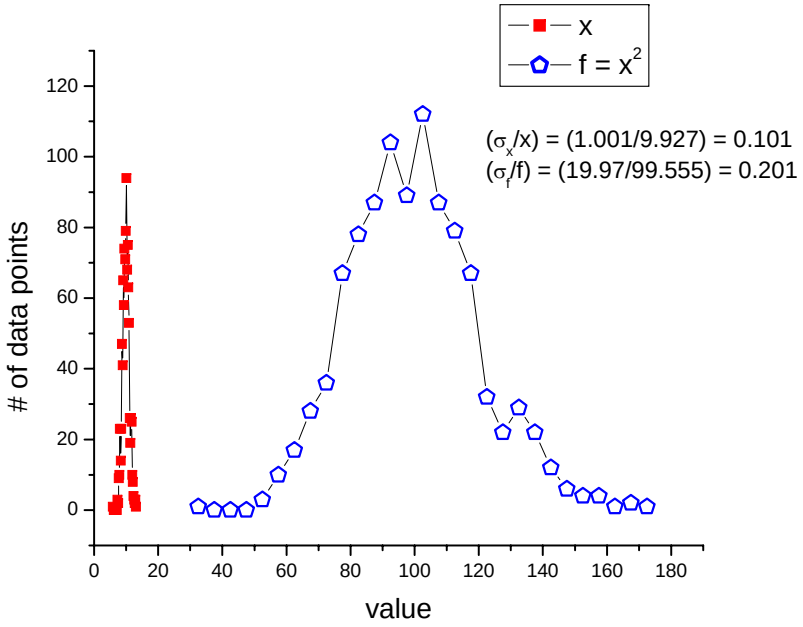
$$\sigma_g^2 = \left|\frac{4\pi^2}{T^2}\right|^2 \sigma_l^2 + \left|2\frac{4\pi^2}{T^3}\right|^2 \sigma_T^2$$

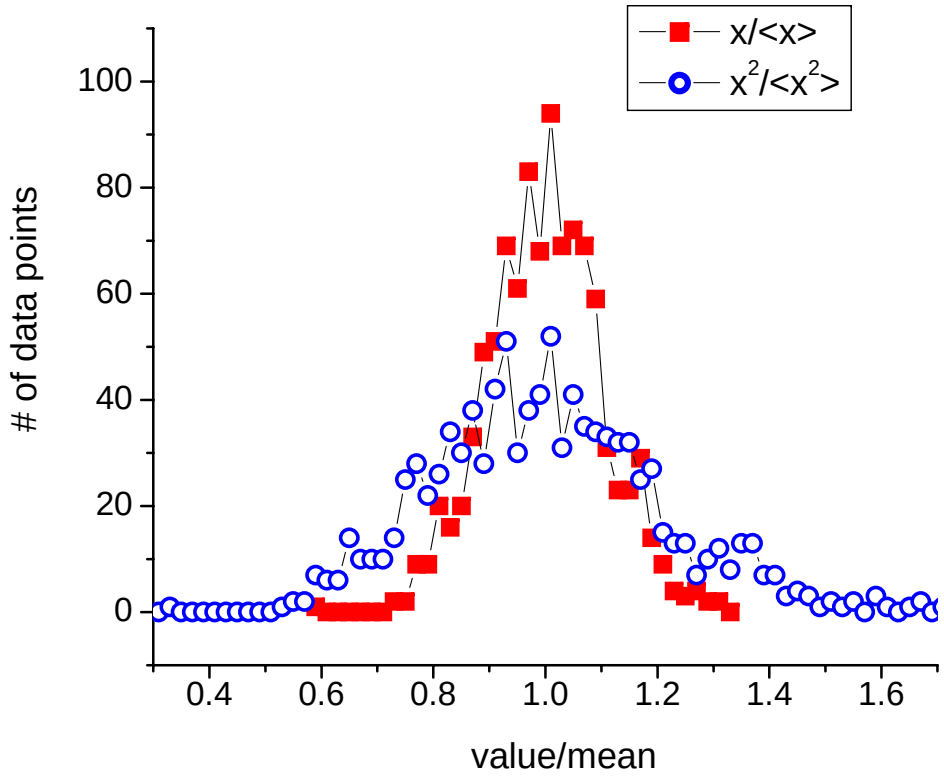
relative

$$\frac{\sigma_g^2}{g^2} = \frac{\sigma_l^2}{l^2} + 2\frac{\sigma_T^2}{T^2}$$

σ for a function of a variable

$f=x^2 \rightarrow \sigma_f / f = 2 * \sigma_x / x$

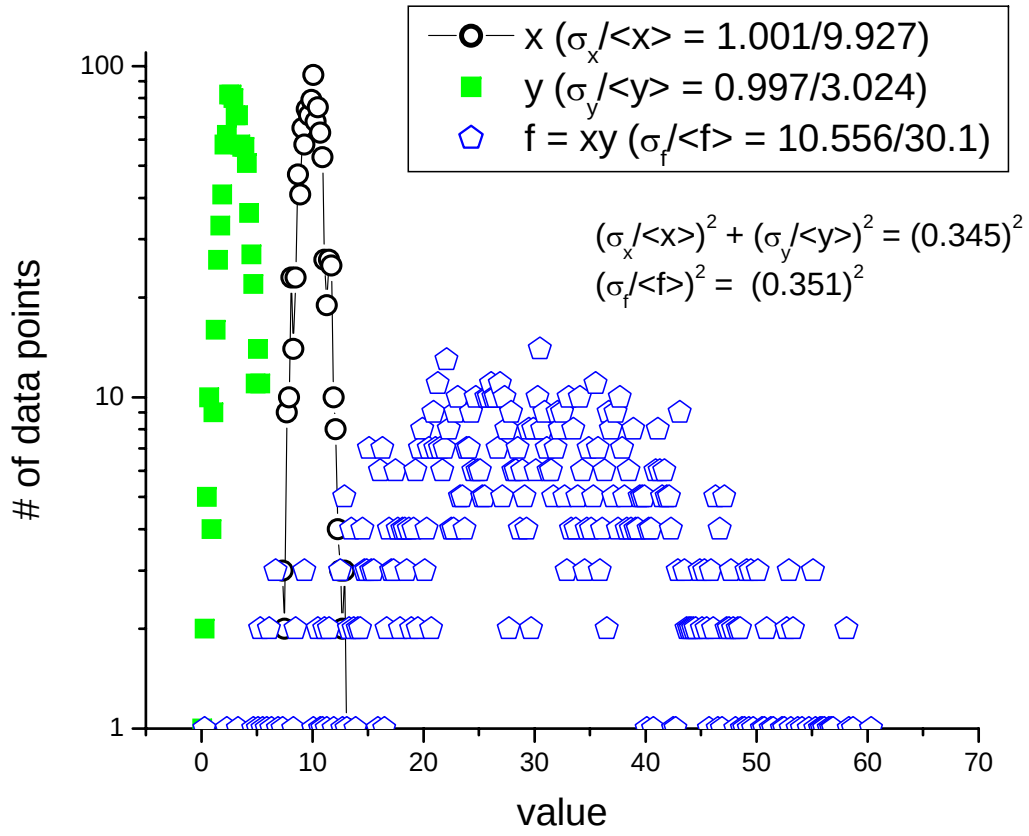




σ for a function of two variables

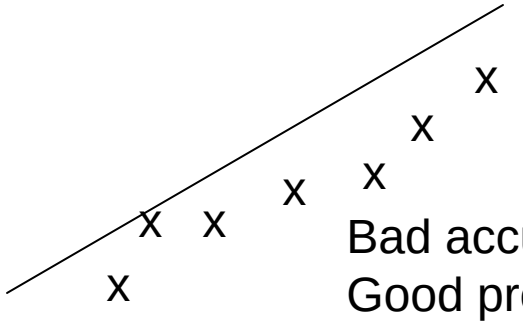
	x	y	$f = x*y$
mean	9.927	3.024	30.09
Std. dev	1.00121	0.99662	10.556
Std err	0.03166	0.03152	0.338

$$f=xy \rightarrow (\sigma_f/f)^2 = (\sigma_x/x)^2 + (\sigma_y/y)2$$



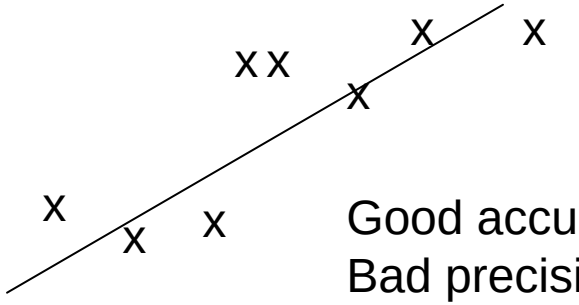
Systematic error vs random errors

Systematic errors:
Reproducible inaccuracies that shift the result consistently in one direction.
Example: Wrong calibrated instrument.
Often one can correct results for systematic errors.



Bad accuracy
Good precision

Random errors:
Statistical fluctuation of measured data due to randomness in the measured process or in the measurement device.
Random errors can be reduced through averaging.



Good accuracy
Bad precision

Reporting results (significant figures)

Round uncertainty to 1-2 significant figures

Quote the result to the same number of decimal places

Good or bad?

$G = 6.67 \times 10^{-11} \pm 5 \times 10^{-13}\ \mathrm{Nm^2kg^{-2}}$

$G = (6.67 \pm 0.05) \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

$G = (6.673 \pm 0.010) \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

$G = (6.67 \pm 0.001) \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

$G = (6.6726 \pm 0.05) \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$